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COMPUTATION OF TURBULENT ASYMMETRIC WAKE

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SUMMARY

The development of asymmetric wake behind an aerofoil in turbulent incompressible flow has been computed using finite volume scheme for solving two-dimensional Navier–Stokes equations along with the k–ε model of turbulence. The results are compared with available experimental data. It is observed that the computed shift of the point of minimum velocity with distance is sensitive to the prescribed value of the normal component of velocity at the trailing edge of the aerofoil. Making the model constant C_μ as a function of streamline curvature and changing the production term in the equation for ε, has only marginal influence on the results.

KEY WORDS Asymmetric wake Turbulent flow k–ε model of turbulence.

INTRODUCTION

Wake is an important flow regime which occurs in a variety of practical situations. Several experimental investigations have been carried out to obtain the mean velocity and turbulent stresses in this flow, e.g. References 1–6. Attempts have also been made to compute these flows, e.g. References 7–9. It is now known that the near wake with no cross-stream pressure gradient can be predicted satisfactorily by parabolic scheme using k–ε model or Reynolds stress model of turbulence. However, wake behind an aerofoil, especially, at an angle of attack is asymmetric and develops in a flow with cross-stream pressure variation. Such a flow cannot be computed by parabolic schemes, and Chang *et al.*[9] suggest that an elliptic scheme will work better. Hence, an investigation to calculate wakes using an elliptic scheme has been initiated. As a first step, symmetric and asymmetric wakes in laminar and turbulent flows with zero pressure gradient were computed and compared with available data to validate the scheme. These results are given in Tulapurkara *et al.*[10] In the present paper, the asymmetric wake developing from the boundary layers at the trailing edge of an aerofoil at an angle of attack of 6° is computed. The results are compared with the experimental data of Ramaprian *et al.*[5] Trials have been done with different distributions of normal velocity component at the trailing edge and with change in the values of the constants used in the model of turbulence.

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COMPUTATIONAL TECHNIQUE

Pun and Spalding[11] have developed a numerical scheme called 2/E/FIX to calculate both laminar and turbulent flows governed by two-dimensional Navier–Stokes equations. It uses staggered grid, upwind differencing and the SIMPLE algorithm of Caretto *et al.*[12] For the calculation of turbulent flows, the k–ε model of Launder and Spalding[13] with standard constants is used. This code has been suitably modified to calculate the development of wakes with prescribed (i) initial profiles of U, V, k and ε and (ii) the free-stream pressure gradient.

Governing equations

The governing equations in tensor notation and incorporating the k–ε model of turbulence are:

$$\partial \bar{U}_i/\partial x_i = 0, \tag{1}$$

$$\frac{\partial}{\partial x_j}[\rho \bar{U}_j \bar{U}_i] = -\frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j}\left[\mu_{\text{eff}}\left(\frac{\partial \bar{U}_i}{\partial x_j} + \frac{\partial \bar{U}_j}{\partial x_i}\right)\right], \tag{2}$$

$$\frac{\partial}{\partial x_j}[\rho \bar{U}_j k] = \frac{\partial}{\partial x_j}\left[\frac{\mu_{\text{eff}}}{\sigma_k}\frac{\partial k}{\partial x_j}\right] + G - \rho\varepsilon, \tag{3}$$

$$\frac{\partial}{\partial x_j}[\rho \bar{U}_j \varepsilon] = \frac{\partial}{\partial x_j}\left[\frac{\mu_{\text{eff}}}{\sigma_\varepsilon}\frac{\partial \varepsilon}{\partial x_j}\right] + \frac{\varepsilon}{k}(C_{\varepsilon 1} G - C_{\varepsilon 2}\rho\varepsilon), \tag{4}$$

where

$$\overline{-u_i' u_j'} = \nu_t\left[\frac{\partial \bar{U}_i}{\partial x_j} + \frac{\partial \bar{U}_j}{\partial x_i}\right] - \frac{2}{3}\delta_{ij}k, \qquad \nu_t = C_\mu k^2/\varepsilon, \qquad \mu_t = \rho\nu_t,$$

$$\mu_{\text{eff}} = \mu + \mu_t, \qquad G = \mu_t \frac{\partial \bar{U}_i}{\partial x_j}\left[\frac{\partial \bar{U}_i}{\partial x_j} + \frac{\partial \bar{U}_j}{\partial x_i}\right] \tag{5}$$

in which $C_\mu = 0{\cdot}09$, $\sigma_k = 1{\cdot}0$, $C_{\varepsilon 1} = 1{\cdot}44$, $C_{\varepsilon 2} = 1{\cdot}92$, $\sigma_\varepsilon = 1{\cdot}3$, and ρ and p are density and pressure, respectively; μ and μ_t are dynamic and eddy viscosities, respectively; $\bar{U}_i$ is mean velocity in x_i-direction; k the turbulent kinetic energy; and ε the rate of dissipation of k. It may be added that in the subsequent discussion, $X = x_1$, $Y = x_2$, $U = \bar{U}_1$, $V = \bar{U}_2$ and $\overline{u'v'} = \overline{u_1' u_2'}$ are also used.

Case considered

The experimental data of Ramaprian *et al.*[5] is used to compare the computed results. In their experiment, a Korn–Garabedian aerofoil was used to generate the asymmetric wake. The chord length c of 914·4 mm and an angle of attack of 6° were selected such that there is no flow separation on the aerofoil and the two boundary layers on either side of the aerofoil near the trailing edge were as different in thickness as possible[5] (18 and 36 mm). The external velocity at the trailing edge was 19·25 m/s but increased to 22·25 m/s at $X = 12{\cdot}7$ mm from trailing edge and then remained constant. This steep change in velocity within $X = 12{\cdot}7$ mm gave computational results which were diverging and after many trials the free-stream velocity of 22·25 m/s was prescribed through out. This velocity is also the reference velocity (U_{ref}) in the experiments.

Boundary conditions

The boundary conditions are to be specified at the edges of the computational domain which extends in the x-direction from the trailing edge of the aerofoil to a point sufficiently far downstream so that the location of the exit plane does not affect the calculated values elsewhere. The upper and lower edges of the domain lie in the potential flow region above and below the wake.

At the top and bottom edges of the wake, the boundary condition is such that the local velocity equals the potential flow velocity. This velocity was chosen equal to the experimental value of 22·25 m/s. The k and ε have zero gradient across the edges; normal component of velocity (V) at these edges is calculated using the continuity equation.

The basic 2/E/FIX code, which computes the flow through a channel, prescribes that V is zero and all other variables have zero streamwise gradient at the exit plane. However, in a wake, the variables continue to change in the streamwise direction even far downstream. But, after some distance behind the body, the second derivative of all the variables in the streamwise direction becomes much smaller than that in the cross-stream direction and, thus, the governing equations reduce from elliptic to parabolic in nature. Hence, the exit boundary condition in the basic code is modified and it is specified that for all variables the second derivative in the streamwise direction is zero there. This requires careful location of the exit plane and in this investigation the experimental data were taken into consideration to arrive at its location. The experimental results[5] show that the point of minimum velocity ($Y_{U_{min}}$) remains the same after $X=1200$ mm from the trailing edge. Secondly, the pressure coefficient also remains almost constant from this station onwards. Hence, the exit plane was taken at 1365 mm. The profile of U at inlet to the computational domain is obtained by smoothing the experimental velocity profiles in the upper and lower boundary layers at the trailing edge. The profile of k is obtained using profile of $\overline{u'_1 u'_2}$ and the expression $k=\overline{u'_1 u'_2}/0{\cdot}3$. The profile of ε is obtained from the following expression:

$$\varepsilon=(0{\cdot}3k)^{1{\cdot}5}/l, \tag{6}$$

where

$$l=0{\cdot}085\delta \tanh\left[\frac{\chi}{0{\cdot}085}\cdot\frac{y}{\delta}\right], \quad \chi=0{\cdot}41$$

and l is the mixing length given by Michel *et al.*[14]

The normal velocity component (V) is evaluated from the flow angles given at the first station downstream of the trailing edge ($X=25{\cdot}4$ mm).

Convergence and grid independence

Values of under-relaxation factors which produced numerical stability were found by trials. Generally, a relaxation factor of 0·5 was used for all the variables, a smaller value of 0·3 was needed in some cases. Convergence was observed by monitoring the residual sources and field values at a specified location. It was achieved after about 900 iterations. In the initial stages of the wake development, the velocity changes rapidly especially in the y-direction. Hence, to avoid steep velocity gradients in these regions, a close grid spacing is required in the central part of the wake. Thus, to obtain a converged solution, a minimum grid size of 128 × 57 grid lines was required to cover the computation domain extending from $X=0$ to $X=1365$ mm and from $Y=-60$ mm to $Y=+250$ mm, X and Y being measured from the trailing edge of the aerofoil. However, to check the grid independence, the computations were also carried out with grids of size 160 × 71 and 192 × 86. Slight difference was found between the profiles obtained with 128 × 57 and 160 × 71 grids but differences between profiles for 160 × 71 and 192 × 86 were not noticeable. Hence, for the subsequent computations, 192 × 86 grid was used. The computations took 11 h of

CPU time on Siemens BS 2000 computer. The same computations when performed on a Parallel Computer (PARAM) at Centre for Development of Advanced Computing, Pune, India, required only 50 min with 64 transputers.

RESULTS AND DISCUSSION

To begin with, the computations were carried out by prescribing $V=0{\cdot}0$ at the inlet to the calculation domain, i.e. at the trailing edge of the aerofoil. The calculated distributions of U, k and $\overline{u'v'}$ are shown along with the experimental profiles in Figure 1(a)–1(c). The variations with X of (i) the ordinate of the point of minimum velocity ($Y_{U_{min}}$), (ii) the wake half-width ($b_{1/2}$) and (iii) the velocity defect ($U_{deficit}$), are shown in Figures 2–4. The profiles of U, k and $\overline{u'v'}$ exhibit trends similar to the experimental data. From Figures 1(a) and 2, it can be seen that the computed shift of the wake centre-line with X is rather small and is in a direction opposite to that in the experiment. Thus, the use of elliptic scheme with $V=0{\cdot}0$ does not give the desired shift in $Y_{U_{min}}$ and computations need to be carried out by prescribing V distribution at the inlet. The experimental data on V at the trailing edge of aerofoil are lacking in Reference 5. Hence, it was decided to calculate V from the distribution of flow angle (θ) given at the nearest station, viz. at $X=25{\cdot}4$ mm. This distribution of V is denoted by V_{25}.

Computations were performed by prescribing V at inlet equal to V_{25} and the results are included in Figures 1(a)–1(c) and 2–4. The results, when compared with experimental data, show that (i) the shift in the point of minimum velocity is in the correct direction, but the shift is larger;

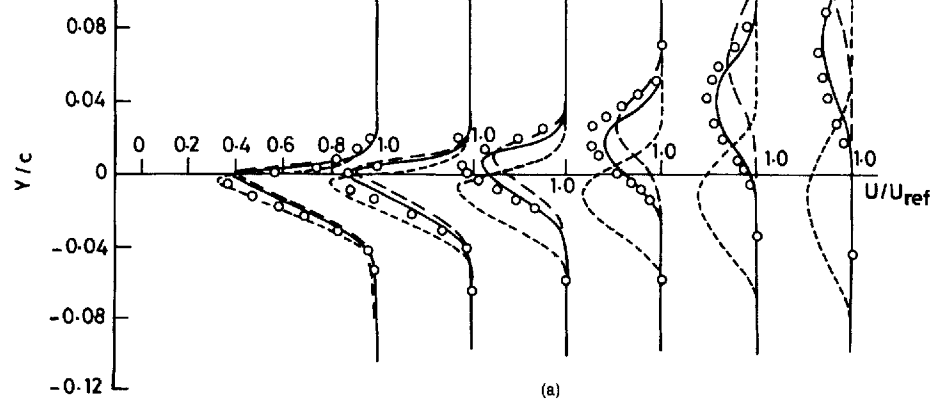
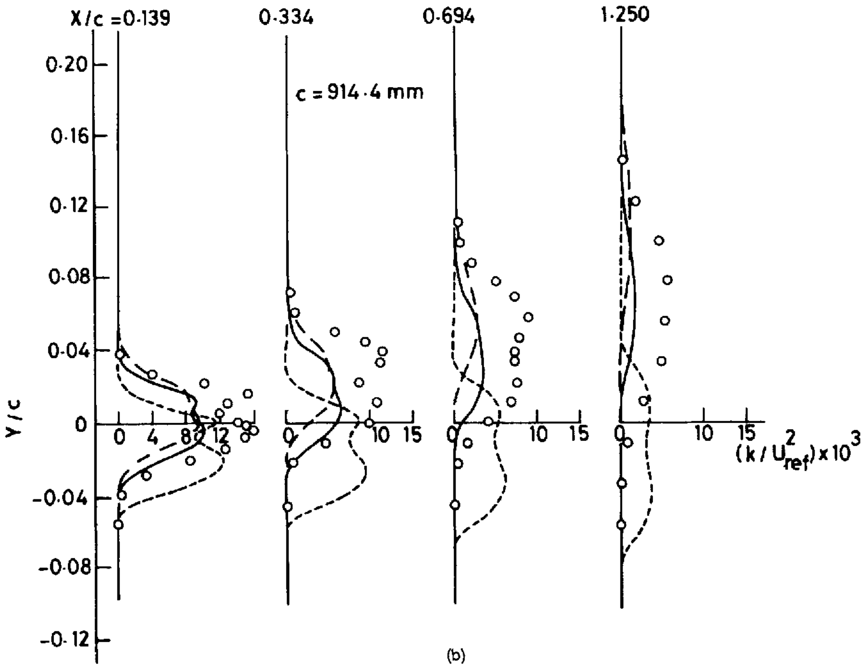
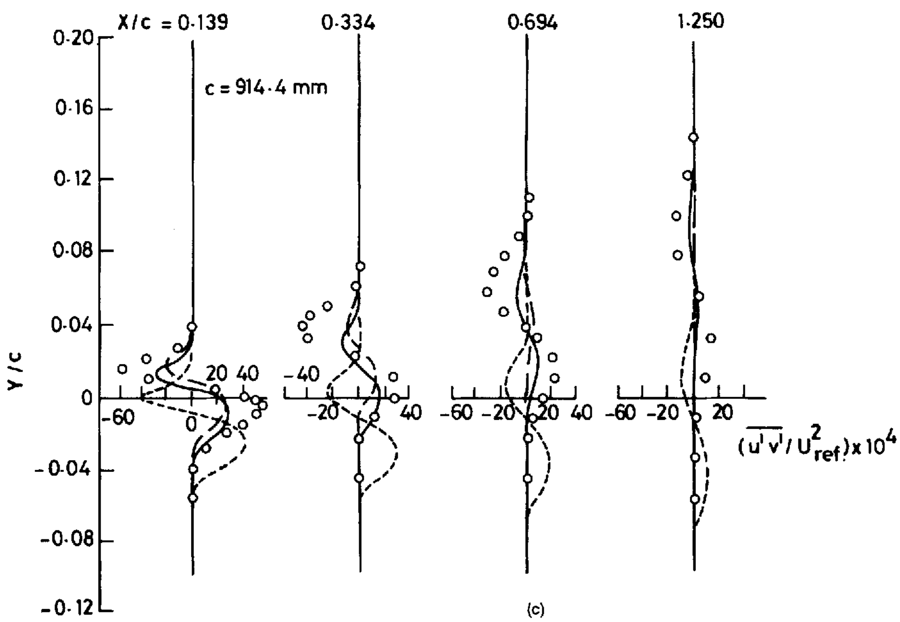


Figure 1. Aerofoil wake computation with standard k–ε model: (a) mean velocity; (b) turbulent kinetic energy; (c) Reynolds shear stress: ○, Experimental Ramaprian *et al.*[5]; – – – – –, $V=0{\cdot}0$; ———, $V=V_{25}$; ———, $V=0{\cdot}75V_{25}$



(b)



(c)

Figure 1. (Continued)

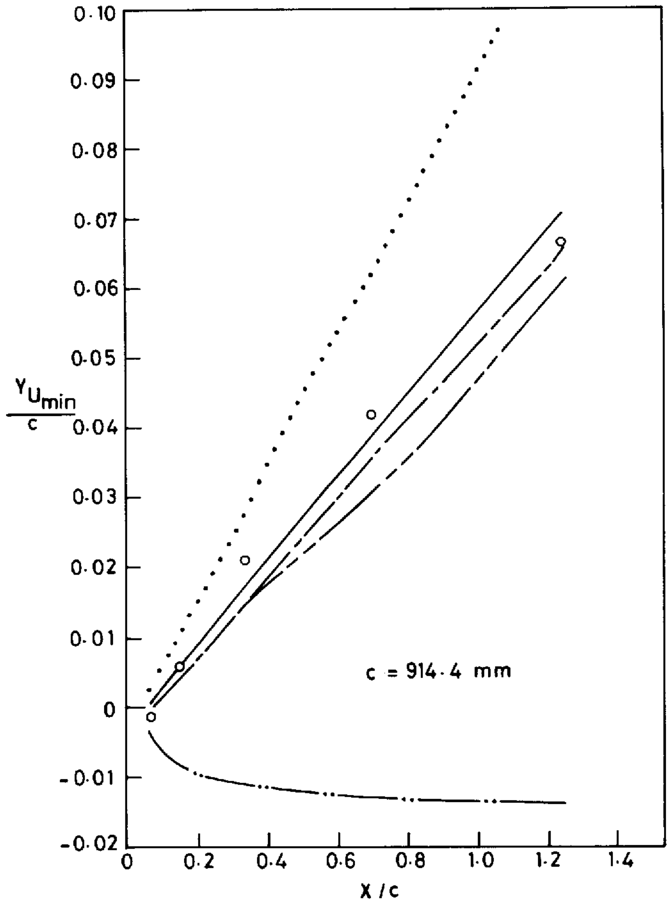


Figure 2. Variation of the point of minimum velocity: ○, Experimental Ramaprian *et al.*[5]; —··—, Standard constants and $V=0$;........, Standard constants and $V=V_{25}$; ——, Standard constants and $V=0{\cdot}75\,V_{25}$; — — —, C_μ alone variable and $V=0{\cdot}75\,V_{25}$; - - -, C_μ and P_ε variable and $V=0{\cdot}75\,V_{25}$

(ii) the predicted half-width of wake ($b_{1/2}$) is larger; (iii) maximum velocity defect (U_{deficit}) is smaller and (iv) computed values of k and $\overline{u'v'}$ are lower as compared to those in the experiment.

Calculations were, therefore, performed by prescribing V at the inlet as F times V_{25} with F varying from 0·5 to 0·75. The results when $V_{\text{inlet}}=0{\cdot}75\,V_{25}$ are also shown in the figures referred above. It is observed that (i) there is a reasonable agreement between the experimental and computed velocity profiles; (ii) the predicted values of wake characteristics like $Y_{U_{\min}}$ and $b_{1/2}$ (Figures 2 and 3) are close to the experimental values; (iii) the calculated profiles of k and $\overline{u'v'}$ show lower values than the experimental ones; (iv) the velocity defect (U_{deficit}) is also lower than the experimental values (Figure 4).

It was felt that these differences in computed and experimental values may be due to the deficiency of the standard k–ε model, and some corrections to the model constants may be required. It is to be noted that the wake of an aerofoil kept at angle of attack develops in a flow with streamline curvature. Keeping this in mind, the following modifications to the standard $k-\varepsilon$

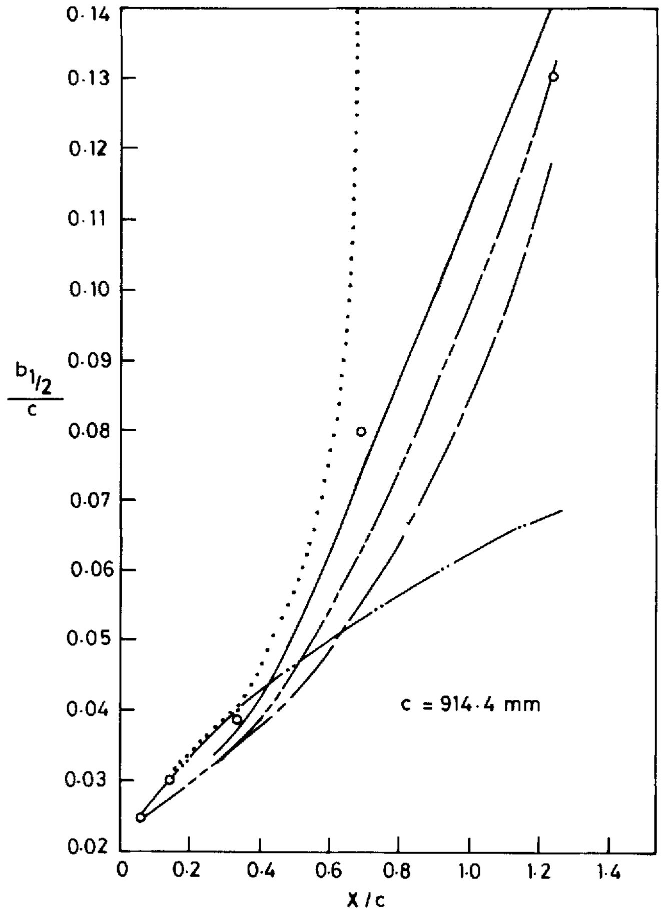


Figure 3. Variation of the wake half-width (symbols same as for Figure 2)

model suggested by Leschziner and Rodi[15] were tried out:

(i) C_μ as a function of curvature, i.e.

$$C_\mu = \max\left[0{\cdot}025,\ \frac{0{\cdot}09}{1+0{\cdot}57\dfrac{k^2}{\varepsilon^2}\left(\dfrac{\partial U_s}{\partial n}+\dfrac{U_s}{R_c}\right)\dfrac{U_s}{R_c}}\right] \tag{7}$$

(ii) modifying the production term in ε equation from

$$P_\varepsilon = C_{\varepsilon 1}\frac{\varepsilon}{k}G \text{ to } P_\varepsilon = \frac{\varepsilon}{k}(C_1 G - C_2 S_{ns}), \tag{8}$$

where, $C_1 = 2{\cdot}24$ and $C_2 = 0{\cdot}8$; s and n are co-ordinates along and perpendicular to the streamline, respectively; U_s the velocity along streamline; R_c the radius of curvature of streamline and S_{ns} the strain rate due to streamline curvature.[15]

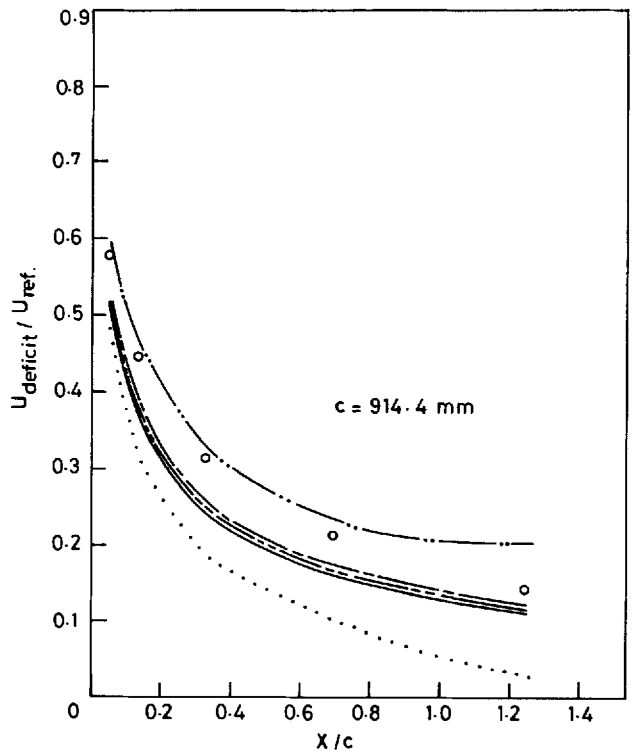


Figure 4. Variation of the maximum deficit velocity (symbols same as for Figure 2)

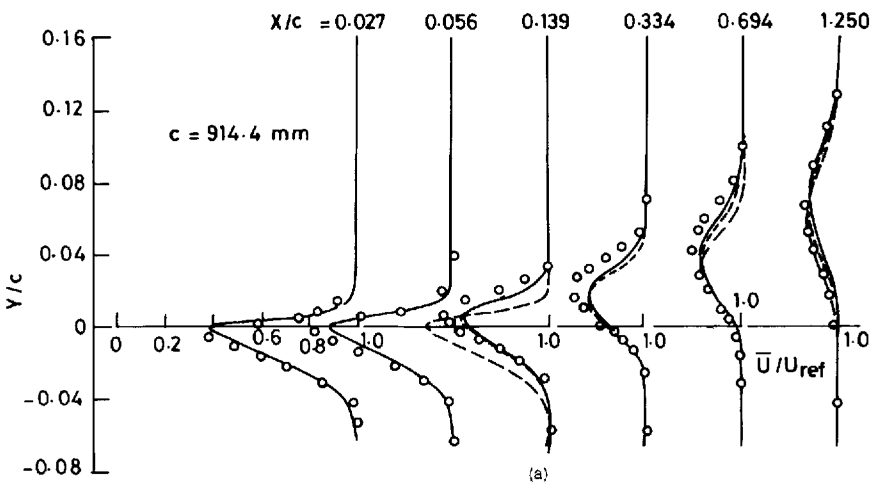
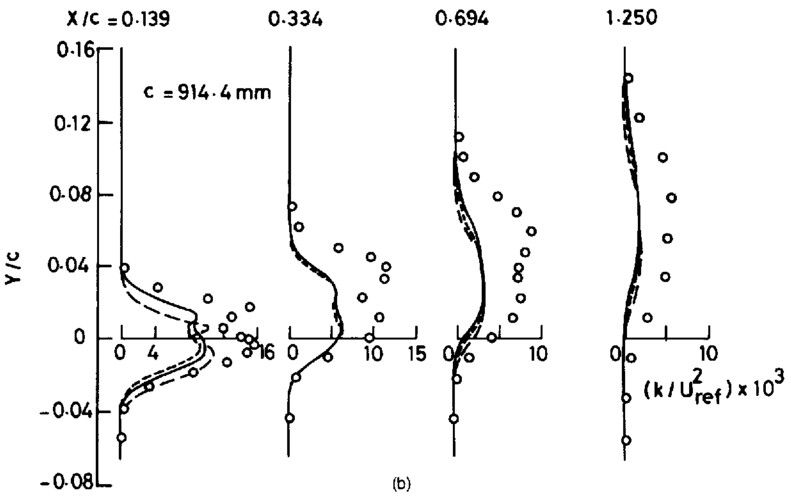


Figure 5. Aerofoil wake computation with $V=0{\cdot}75V_{25}$ and influence of turbulence model variations (a) mean velocity, (b) turbulent kinetic energy, (c) Reynolds shear stress: ○, Experimental Ramaprian *et al.*[5]; ——— Standard k–ε model; – – – – –, C_μ alone variable; —— ——, C_μ and P_ε variable



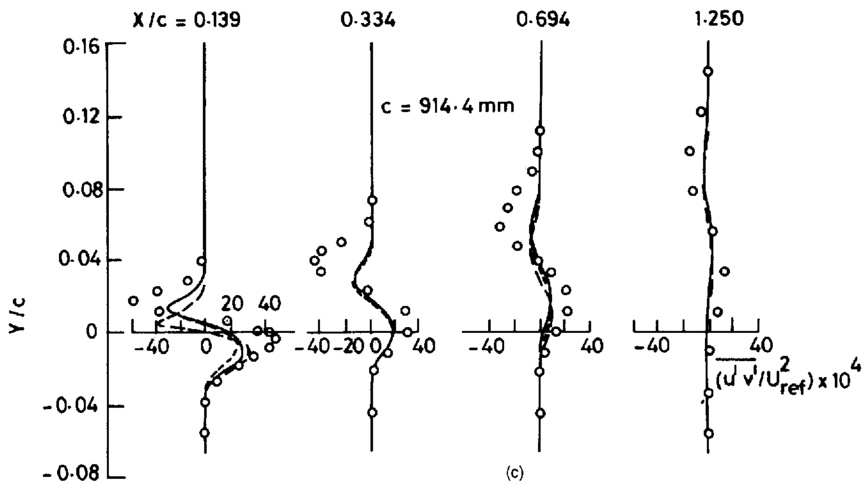


Figure 5. (Continued)

The computed profiles of U, k and $\overline{u'v'}$ in the two cases, viz., (i) with C_μ alone as a function of curvature and (ii) with modifications in both C_μ and P_ε are shown in Figure 5(a)–5(c), respectively. The variations of $Y_{U_{min}}$, $b_{1/2}$ and $U_{deficit}$ for these cases are included in Figures 2–4. It is seen that modifying C_μ alone as a function of a local streamline curvature improves the results but only marginally. The results do not show significant improvement when changes in P_ε term are also incorporated in the model. To investigate this lack of improvement, the curvature parameter $(b_{1/2}/R_c)$, was estimated. At a distance of about 500 mm behind the aerofoil, $b_{1/2}$ is 52 mm and R_c is 140 m. Thus, the curvature parameter is $3{\cdot}7 \times 10^{-4}$. The effect of curvature parameter on the development of a wake has been studied experimentally by Ramjee *et al.*[16] The values of $b_{1/2}/R_c$ were 0·028 and 0·014 in their study. For the latter case, the effect of curvature on wake was small. This explains why, in the present case, where $b_{1/2}/R_c$ is still smaller, the results are insensitive to changes in the model of turbulence.

CONCLUSION

An elliptic scheme with standard k–ε model and proper distribution of normal velocity (V) at the inlet is able to predict the velocity profiles and shift in the location of the point of minimum velocity ($Y_{U_{min}}$) almost correctly. However, the actual direction of the shift in $Y_{U_{min}}$ observed in different investigations,[2,5,17] depends on the angle of attack of the wake producing body and, in turn, on the direction of the V-component of velocity at the trailing edge of the body. Proper prescription of this information is crucial for correct prediction of shift in $Y_{U_{min}}$. Making the model constant C_μ variable and changing the production term P_ε in ε equation have marginal influence on the results in this case.

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